



MEERUT INSTITUTE OF ENGINEERING AND TECHNOLOGY
NH-58, Delhi-Roorkee Highway, Baghpat Road, Meerut – 250 005 (U.P.)

Roll No. 014

Sessional Examination/Class Test – 2 (ODD Semester, 2022 – 23)

Course/Branches: B Tech (CSE/IT/CS-IT/CS-DS/CS-IOT/AI/AIML)

Semester: III

Subject Name: Discrete Structures & Theory of Logic

Max. Marks: 60

Subject Code: KCS – 303

Time Allowed: 120 Minutes

CO - 3: The student will be able to apply concept such as posets, lattices, and also properties of Boolean algebras to study some practical problems concerning computer science.

CO - 4: The student will be able to apply fundamental concepts of mathematical logic to study some practical problems concerning computer science.

Section – A (CO -3)

NOTE: Attempt ALL questions.

(30 Marks)

Q 1 Attempt any six parts of this question. Each part is for two marks.

(6 × 2 = 12 Marks)

- (a) What is a *poset*, i.e., a partially ordered set? (K₁)
- (b) What relation will make the power set $P = \mathcal{P}(\{a, b, c\})$ a poset? (K₁)
- (c) State the steps to obtain a Hasse diagram from the digraph of a poset. (K₁)
- (d) Draw the Hasse diagram of poset $A = \{3, 4, 12, 24, 48, 72\}$, ordered by relation “divides”. (K₂)
- (e) Define maximal element and minimal element of a poset. (K₁)
- (f) What are maximal and minimal elements of $P = \{2, 4, 5, 10, 12, 20, 25\}$, partially ordered by the relation “divides”. (K₂)
- (g) Define *least upper bound* and *greatest lower bound* of a subset of a poset. (K₁)

Q 2 Attempt any three parts of this question. Each part is for six marks.

(3 × 6 = 18 Marks)

- (a) Let \mathbb{Z} denote the set of integers. Show that the relation

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid b = a^k, \text{ for some positive integer } k\}$$

is a partial order on \mathbb{Z} .

(K₃)

- (b) Let D_n denotes the set of divisors of the natural number $n > 1$. Show that it is a partial order set with respect to the relation “divides”. Also draw the Hasse diagram of the poset D_{36} . (K₃)

- (c) For every pair of elements a, b in the poset D_{42} , find $\sup\{a, b\}$. (K₃)

- (d) Give an example of a *finite poset* that neither has a maximal element nor a minimal element. (K₃)

- (e) Consider the set $P = \{1, 2, 3, 4, 5, 6, 7, 8\}$, partially ordered by the relation “divides”. Let $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Find $\sup(A)$, $\inf(A)$, $\sup(B)$, and $\inf(B)$. (K₃)

Section – B (CO -4)

NOTE: Attempt ALL questions.

(30 Marks)

Q 3 Attempt any six parts of this question. Each part is for two marks.

(6 × 2 = 12 Marks)

- (a) Suppose $p \rightarrow q$ is a false statement. Find the truth value of the formula $(\neg p \vee \neg q) \rightarrow q$. (K₁)
- (b) What is the truth tables for the the formula $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$. (K₂)
- (c) What are the *converse* and *contrapositive* of the statement: *If it rains, then I will not go to market.* (K₂)
- (d) Test whether the formula $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology or a contradiction. (K₂)
- (e) What are *inconsistent* premises? (K₁)
- (f) What are *valid argument* and *valid conclusion*. (K₁)
- (g) State *Modus Ponens* and *Modus Tollens*. (K₂)

Q 4 Attempt any three parts of this question. Each part is for six marks.

(3 × 6 = 18 Marks)

- (a) Give the truth tables of the formula $((p \rightarrow q) \vee r) \vee (p \rightarrow q \rightarrow r)$. (K₃)
- (b) Test whether $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \vee r)$ is a *tautology*. (K₃)
- (c) Are the premises $p \rightarrow q$, $p \rightarrow r$, $q \rightarrow \neg r$, p inconsistent? Explain. (K₃)
- (d) Use *direct method* to derive the implication $p \rightarrow q$, $q \rightarrow r$, $p \Rightarrow r$. (K₃, K₄)
- (e) Verify validity of the following conclusion: *If the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police will be happy. The police force is never happy. Therefore, the races are not fixed.* (K₄)